Predictive Recirculation: A model of encoding and replay of sequences in a recurrent neural network

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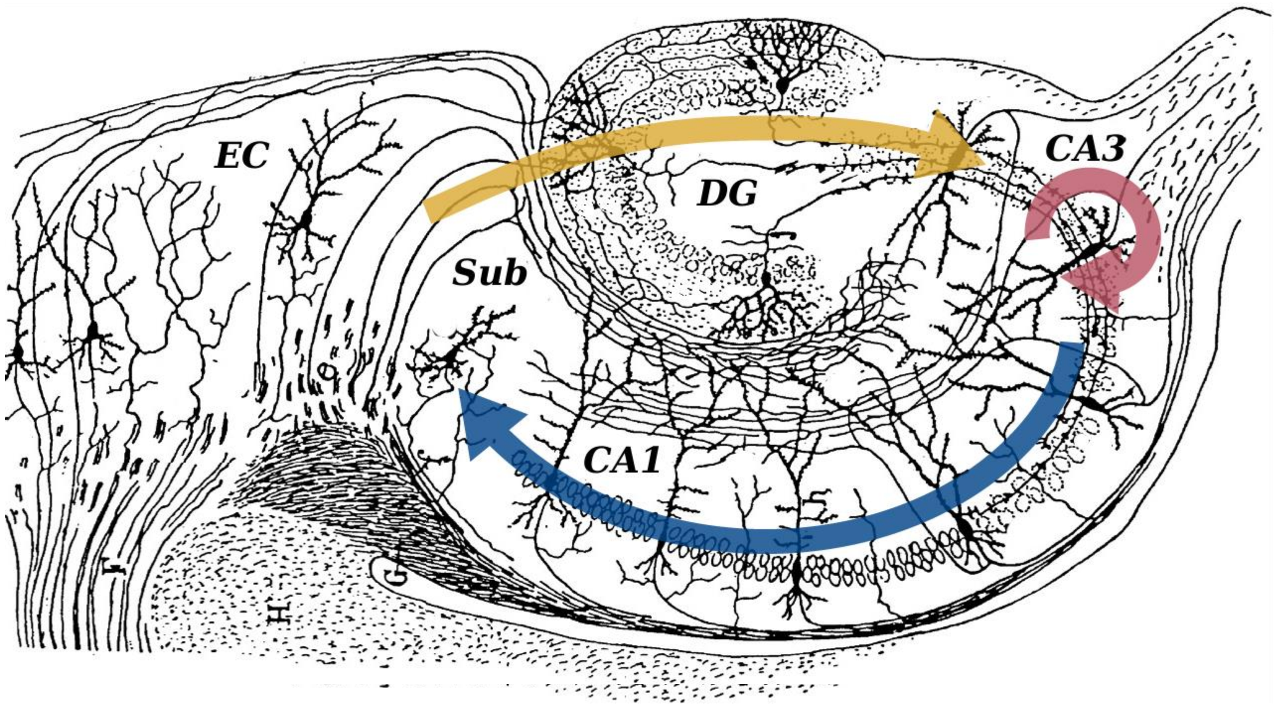
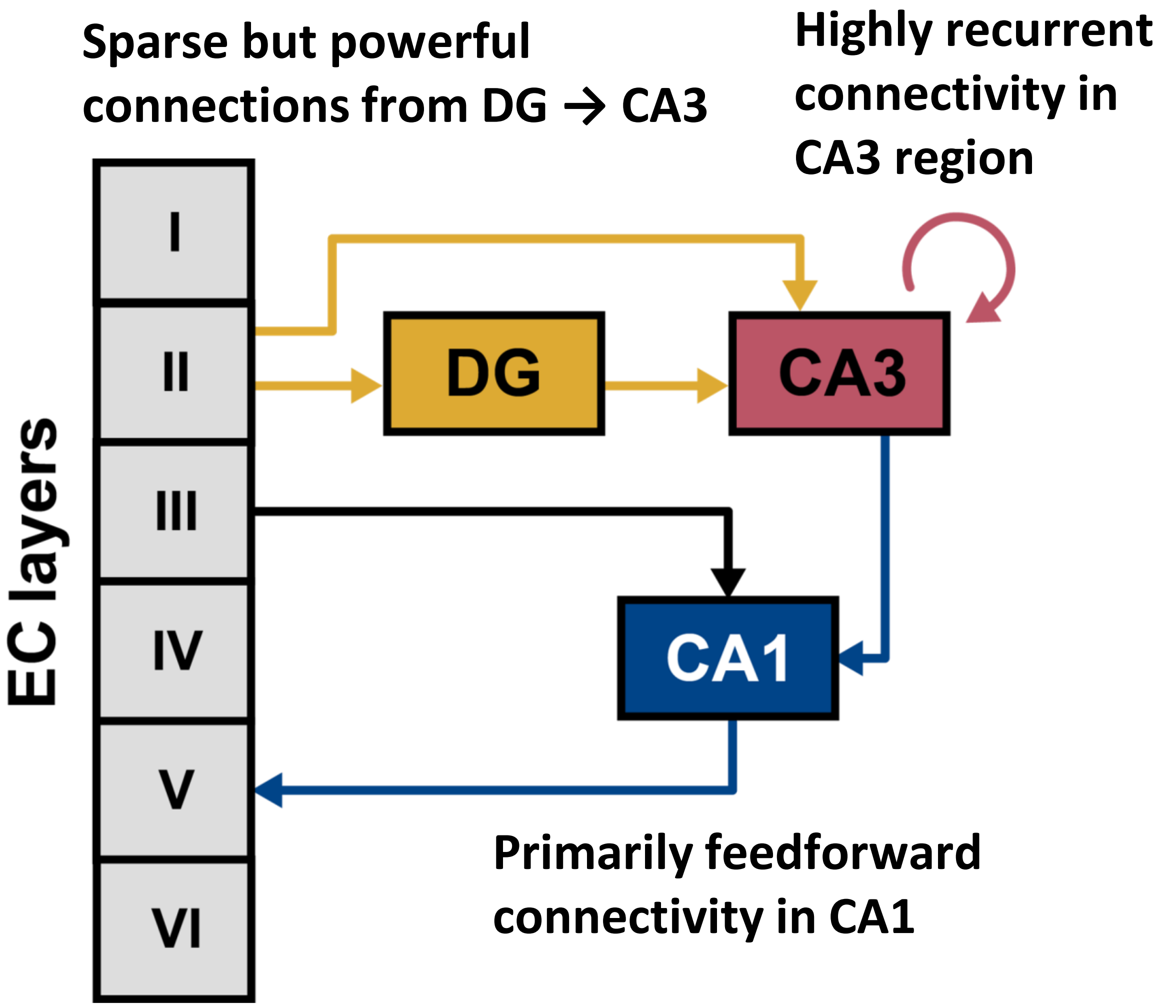
What are the computational processes underlying sequence storage and replay in the hippocampus?

Specialized connectivity patterns at different regions may facilitate different types of computation



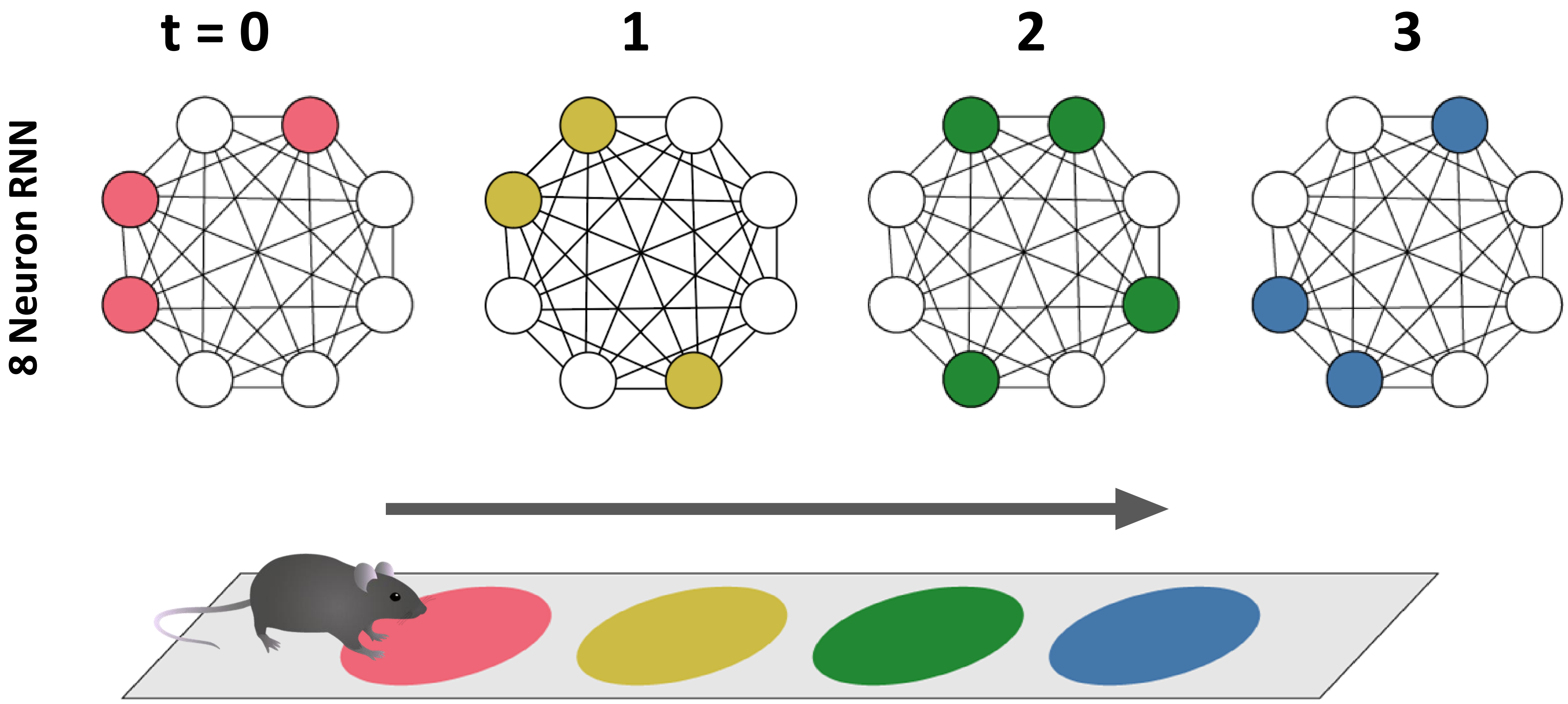


CA3 may store sequences in a recurrent network

Modelling CA3 as an artificial Recurrent Neural Network (RNN)

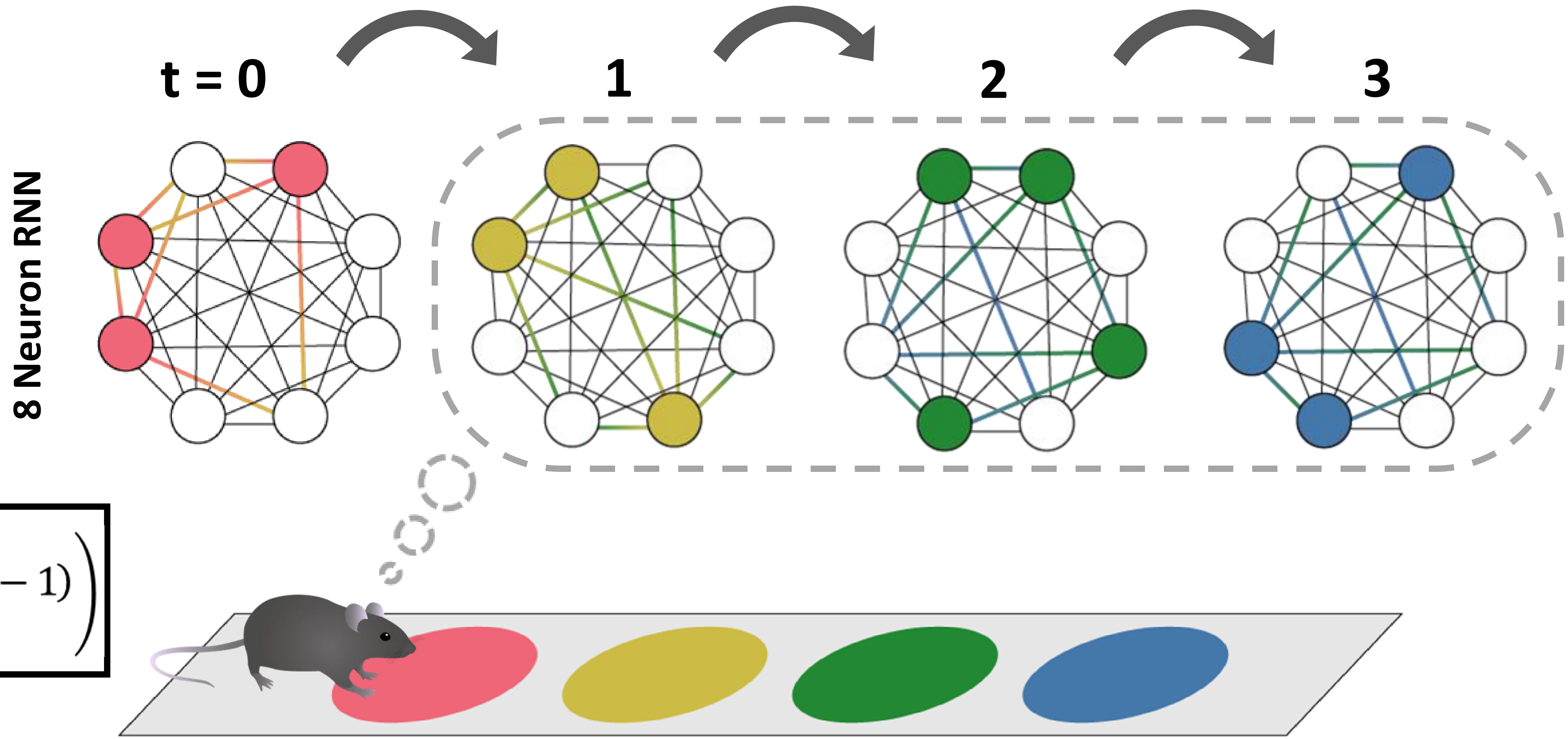
Each element of a sequence is represented as a static pattern in the RNN

During learning, each pattern is an external input to the network



Asymmetric connectivity allows the network to recall the entire sequence, given the first element

During recall, patterns are sequentially activated by recurrent network weights



Network dynamics:

$$x_i(t) = \sigma\left(\sum_j W_{ij}\, x_j(t-1)\right)$$

How can these synaptic weights be learned in a biologically plausible way?

Deriving a local recurrent learning rule:

Network dynamics:

$$\hat{x}_i(t) = \sigma\left(\sum_j W_{ij}\, x_i(t-1)\right)$$

$$\Delta W_{ij} = -\lambda \frac{\partial E_i(t)}{\partial W_{ij}}$$

Gradient descent at each time step

Error function:

$$E_i(t) = \frac{1}{2}\left(x_i(t) - \hat{x}_i(t)\right)^2$$

$$\frac{\partial E_i(t)}{\partial W_{ij}} = \left(x_i(t) - \hat{x}_i(t)\right)\frac{\partial \hat{x}_i(t)}{\partial W_{ij}}$$

$$= \left(x_i(t) - \hat{x}_i(t)\right)\sigma'\left(\hat{x}_i(t)\right)\left(x_j(t-1) + \sum_{k\neq j} W_{ik}\frac{\partial x_k(t-1)}{\partial W_{ij}}\right)$$

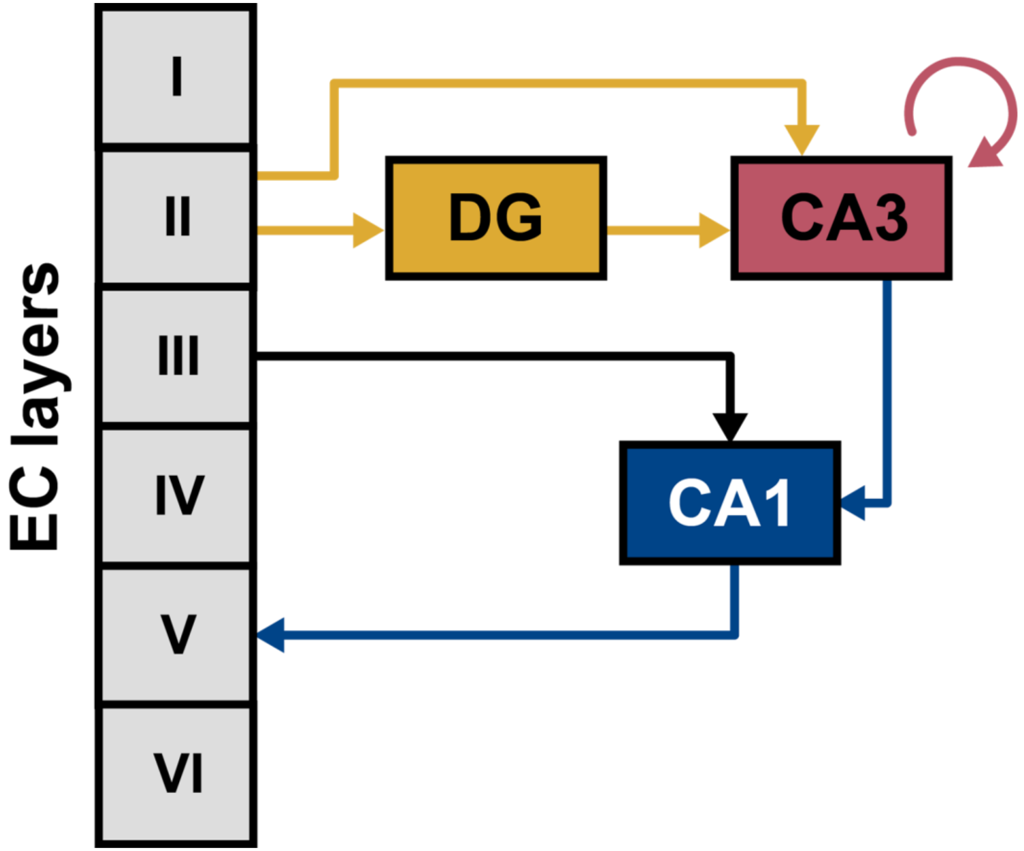
Restricted to spatially and temporally local information:

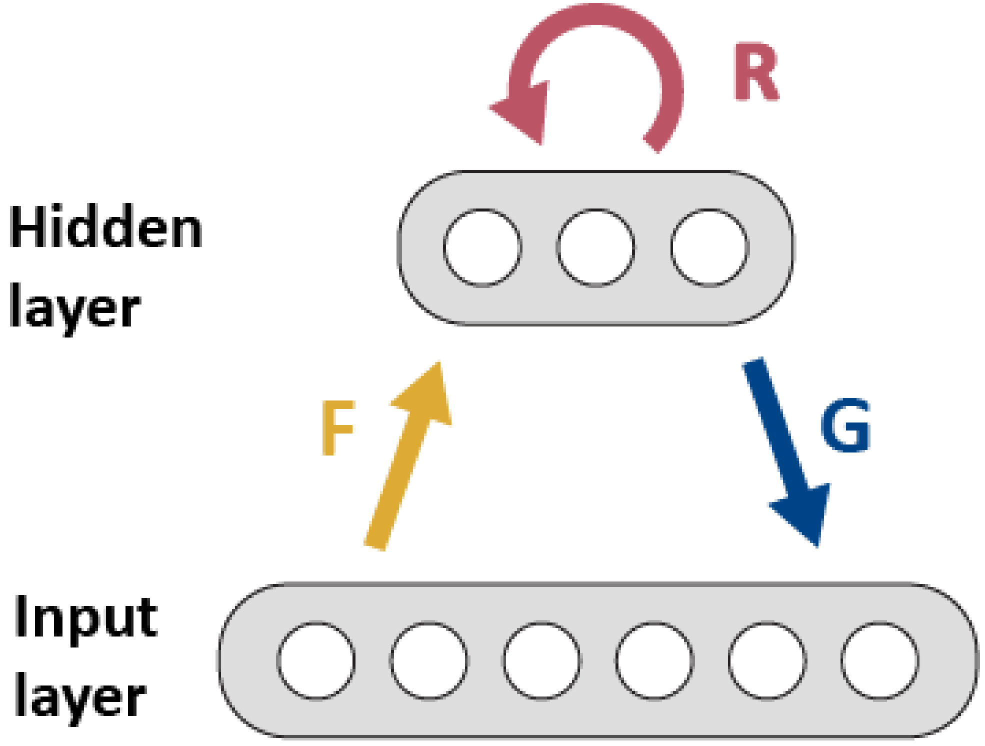
$$= \left(x_i(t) - \hat{x}_i(t)\right)\sigma'\left(\hat{x}_i(t)\right) x_j(t-1)$$

Recurrent Learning Rule:

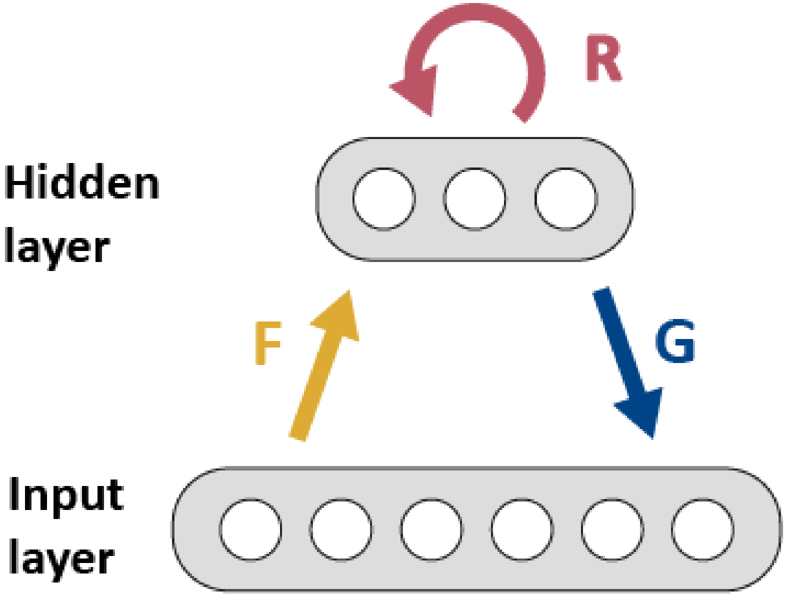
$$\Delta W_{ij} = -\lambda\left(\mathrm{x_i}(\mathrm{t}) - \hat{x}_i(t)\right)\sigma'\left(\hat{x}_i(t)\right) x_j(t-1)$$

Sequences may be recoded to circumvent locally restricted recurrent learning rule

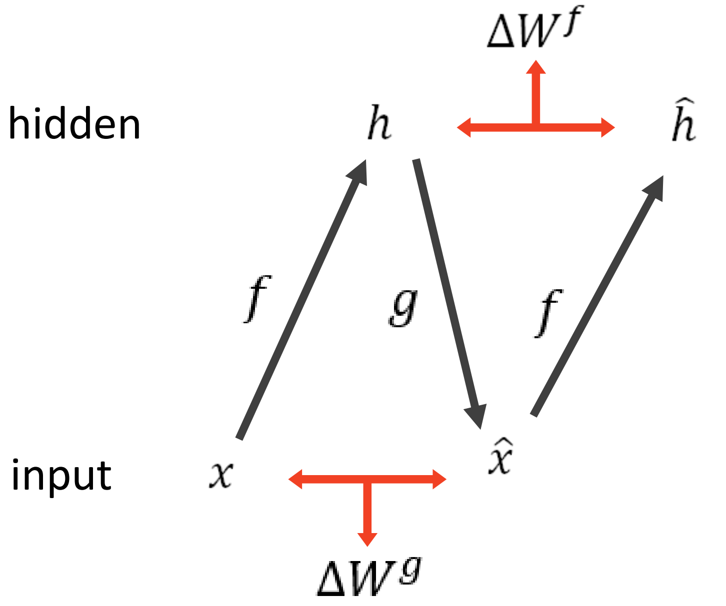




Predictive recirculation combines an autoencoder and RNN



Recirculation autoencoder learning procedure:



Network dynamics:

Synaptic input to unit i

$$s_i = \sum_j W_{ij} u_j$$

Logistic activation

$$u_i = \sigma(s_i) = \frac{1}{1 + e^{-s_i}}$$

Temporal regression

$$u_i^{t+1} = \lambda u_i^t + (1 - \lambda)\sigma(u_i^{t+1})$$

Recirculation Learning Rule

Visible-to-hidden

$$\Delta W_{ij}^f = \epsilon \hat{x}_j (h_i - \hat{h}_i)$$

Hidden-to-visible

$$\Delta W_{ij}^g = \epsilon h_j (x_i - \hat{x}_i)$$

Hinton, G. E., & McClelland, J. (1987). Learning representations by recirculation. In *Neural information processing systems*.

Combined Learning Procedure:

Recurrent Learning Rule:

$$\Delta W_{ij}^{r} := \epsilon\left(h_i^t - p_i^t\right) h_j^{t-1}$$

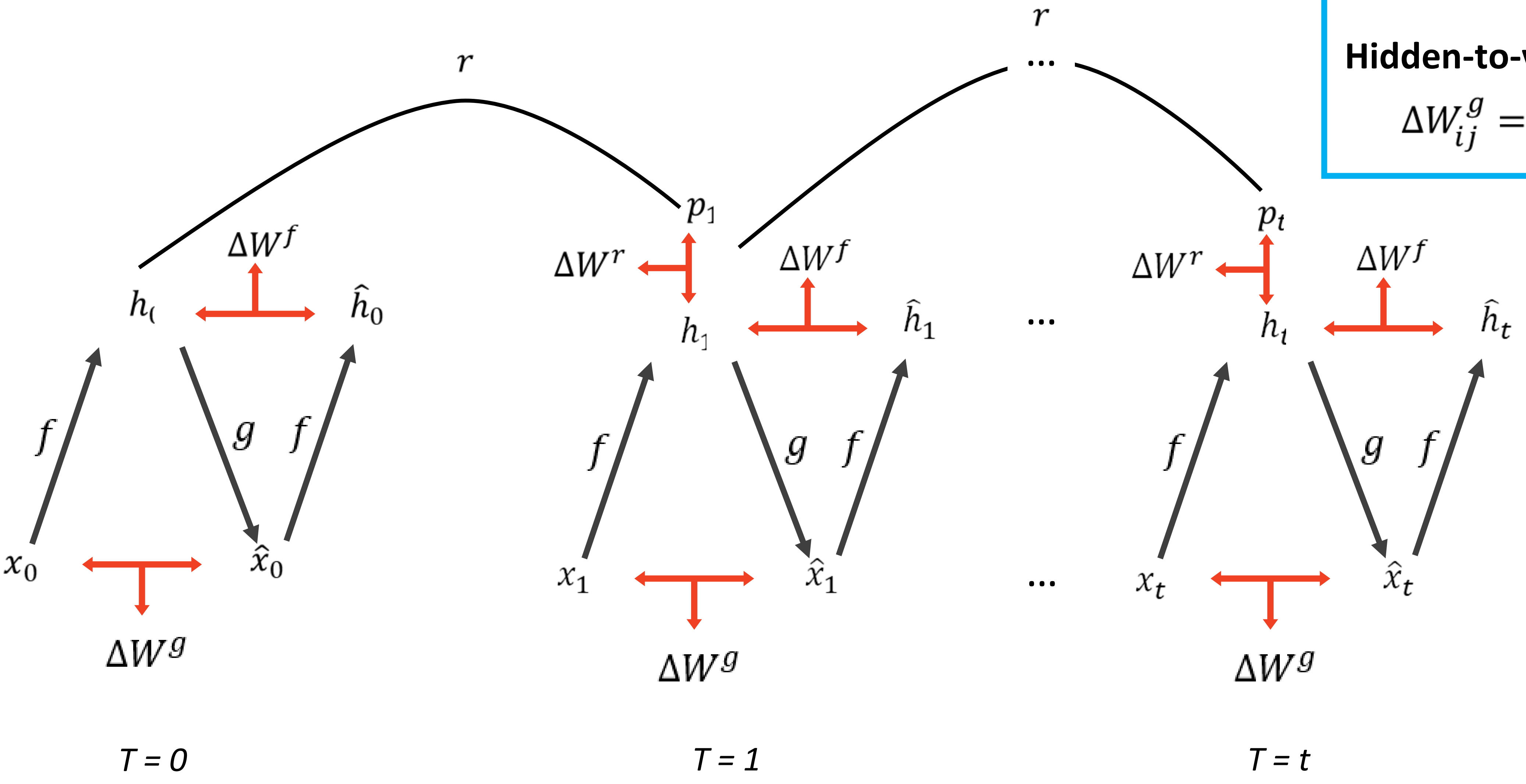
Recirculation Learning Rule

Visible-to-hidden

$$\Delta W_{ij}^{f} = \epsilon \hat{x}_j (h_i - \hat{h}_i)$$

Hidden-to-visible

$$\Delta W_{ij}^{g} = \epsilon h_j (x_i - \hat{x}_i)$$



Recalling a sequence, given the first element:

Recurrent Learning Rule:

$$\Delta W_{ij}^{r} \coloneqq \epsilon\left(h_i^t - p_i^t\right) h_j^{t-1}$$

Recirculation Learning Rule

Visible-to-hidden

$$\Delta W_{ij}^{f} = \epsilon \hat{x}_j (h_i - \hat{h}_i)$$

Hidden-to-visible

$$\Delta W_{ij}^{g} = \epsilon h_j (x_i - \hat{x}_i)$$

$$\begin{tikzcd}
p_t \arrow[r, "r"] & p_{t+1} \arrow[r, "r"] \arrow[d, "g"'] & p_{t+2} \arrow[r, dashed] \arrow[d, "g"'] & {} \\
x_t \arrow[u, "f"] & \hat{x}_{t+1} & \hat{x}_{t+2} &
\end{tikzcd}$$

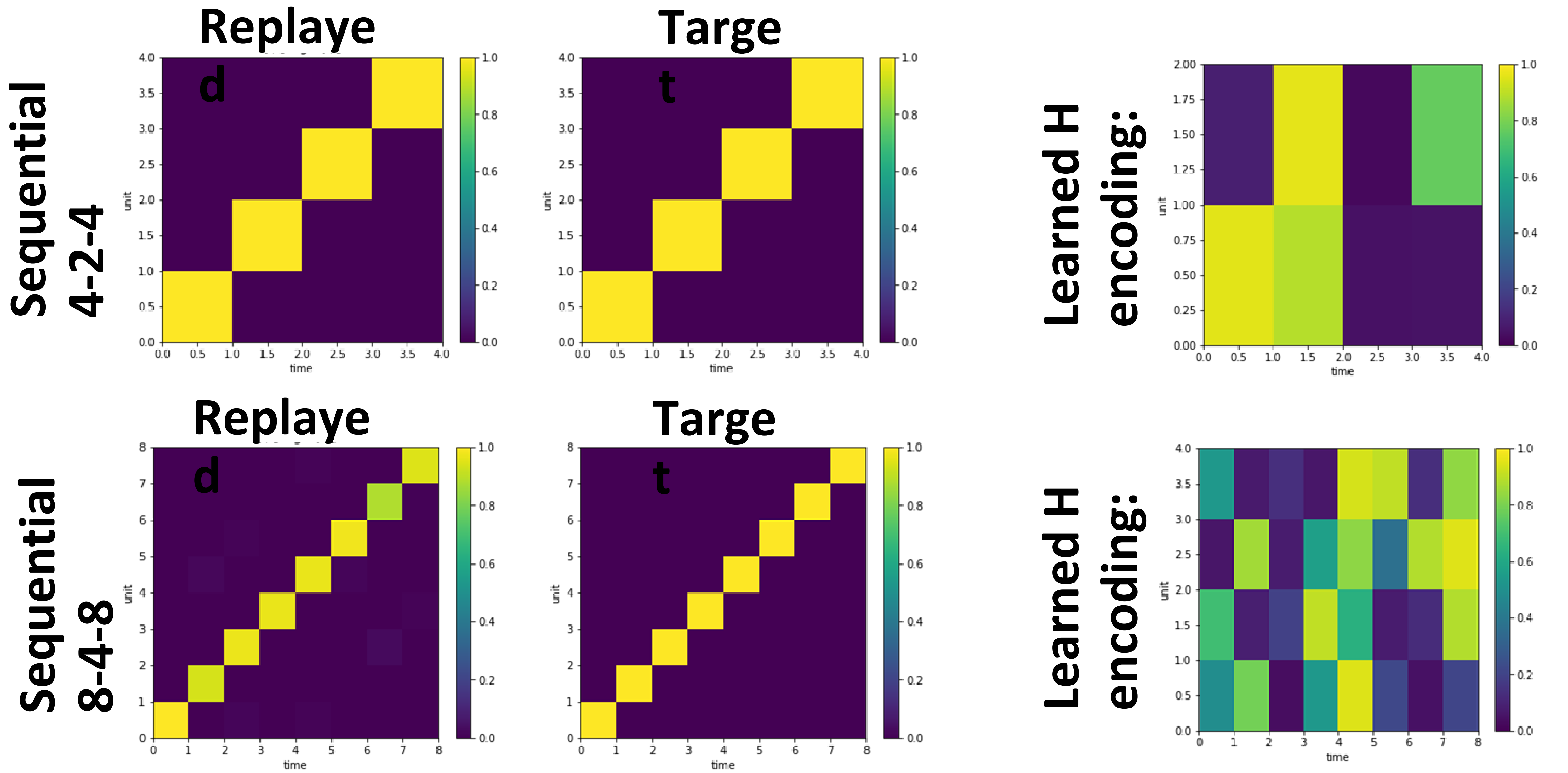
Network dynamics:

$$p_i^t = \sigma\left(\sum_j W_{ij}^{f} x_j^t\right)$$

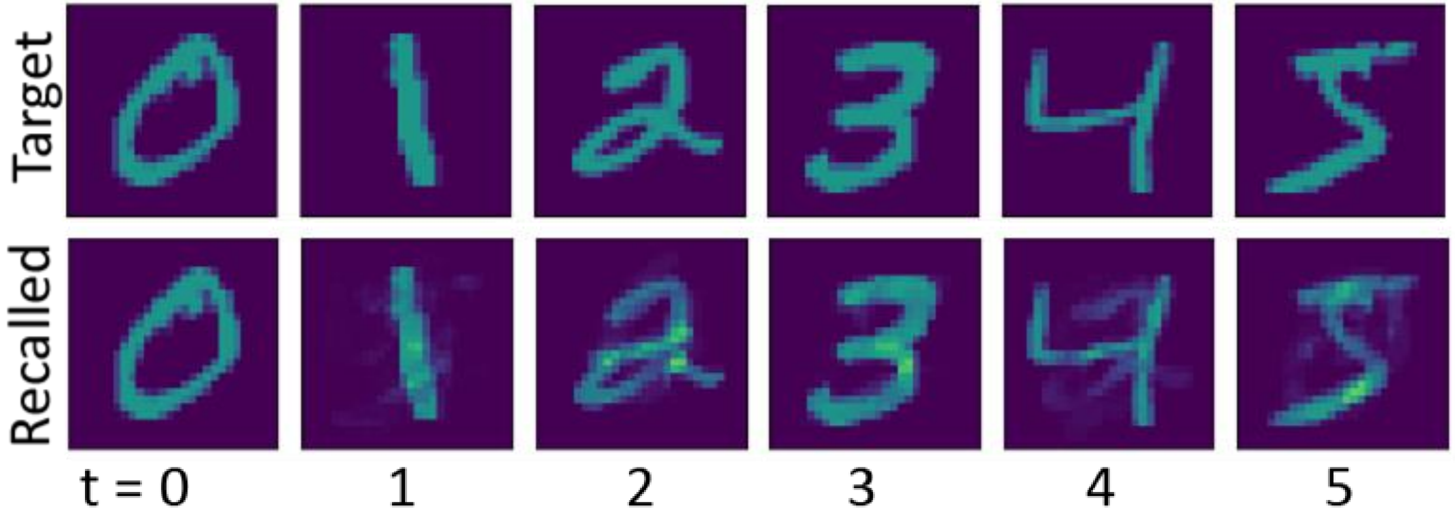
$$\hat{x}_i^t = \sigma\left(\sum_j W_{ij}^{g} p_j^t\right)$$

$$p_i^{t+1} = \sigma\left(\sum_j W_{ij}^{g} p_j^t\right)$$

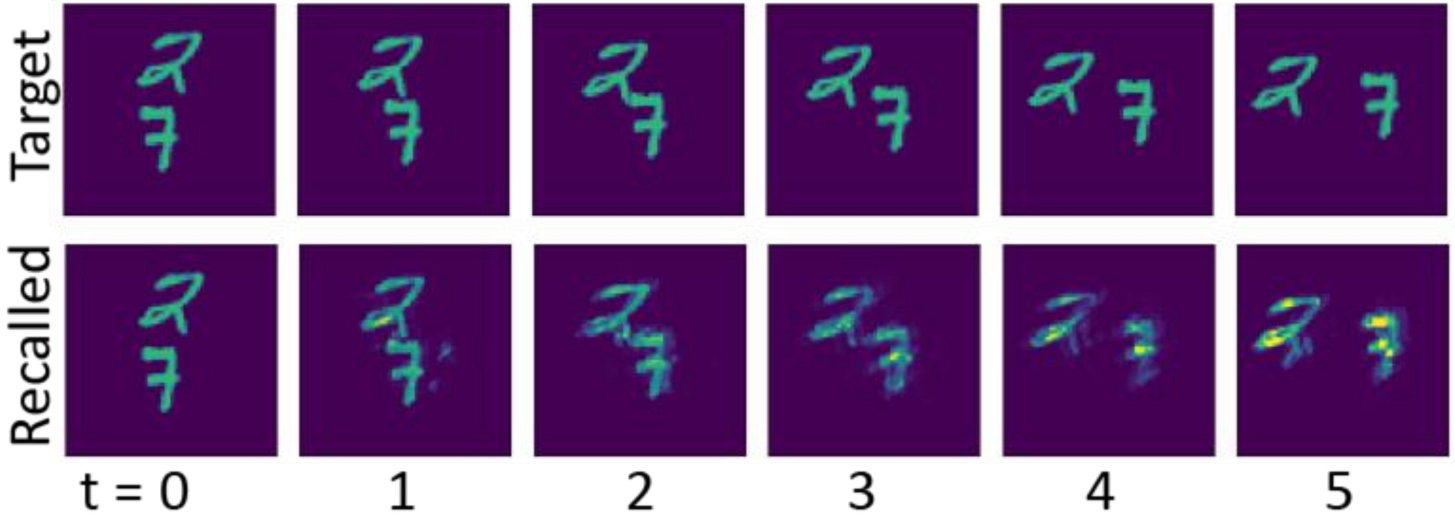
Binary sequences can be encoded and replayed



Sequential MNIST



Moving MNIST



Sequences with high overlap between elements can also be learned

Summary

- Predictive Recirculation provides a biologically plausible model of sequence storage and replay in the hippocampus
- Encoder and recurrent learning rules are temporally and spatially local
- Recoding at different regions may provide a mechanism for circumventing local information restrictions in the recurrent network

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